

MA 8109 SUMMARY LEC 19

Ito-diffusion

$$X_t^x = x + \int_0^t b(X_s^x) ds + \int_0^t \sigma(X_s^x) dB_s, \quad t > 0, \quad [b, \sigma \text{ Lipschitz}]$$

Generator A

$$Af(x) \stackrel{\text{DEF}}{=} \lim_{t \rightarrow 0} \frac{E f(X_t^x) - f(x)}{t} \stackrel{\text{THM}}{=} \sum_i b_i(x) f_{x_i}(x) + \sum_{i,j} \frac{1}{2} (\sigma \sigma^T)_{ij}(x) f_{x_i x_j}(x)$$

Dynkin's formula: $f \in C_c^2$, $E\tau < \infty$, τ st. time

$$E f(X_\tau^x) = f(x) + E \int_0^\tau (Af)(X_s^x) ds$$

Exit time: $\tau_u^x = \inf \{ t > 0 : X_t^x \notin U \}$, $x \in U$ open